



MATHEMATICS SPECIALIST : UNITS 1 & 2, 2023

Tuesday 14th March (T1W7)

Test 1 – Proofs and Counting Techniques (10%)

2.3.1, 1.1.1, 1.1.3 – 1.1.5, 1.2.1 – 1.2.9

Time Allowed 30 minutes	First Name	Surname	Marks / 29 marks
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Circle your Teacher's Name:

Ms Chua

Mr Koulianos

Mr Liu

Ms Robinson

Assessment Conditions: (N.B. Sufficient working out must be shown to gain full marks)

- ❖ Calculators: Not Allowed
- ❖ Formula Sheet: Provided
- ❖ Notes: Not Allowed

PART A – CALCULATOR FREE

Question 1 (6 marks)

Consider the conjecture below:

$$\text{If } y^4 < 2xy^2 - x^2, \text{ then } x \notin \mathbb{R} \text{ or } y \notin \mathbb{R}.$$

a) Write the converse, inverse and contrapositive of this conjecture in the space provided.

(3 marks)

Converse	
Inverse	
Contrapositive	

b) Prove that the contrapositive statement is true.

(3 marks)

Question 2 (5 marks)

Prove the following conjecture:

For $x \in \mathbb{Z}$, $x^2 - 11x + 28$ is never prime.

Question 3 (3 marks)

A set of cards is numbered with all the multiples of 6 between 200 and 900 inclusive. Determine the minimum number of cards which must be selected to ensure that at least 8 cards in the selection have the same last digit. Justify your answer.

Question 4 (6 marks)

a) How many subsets, excluding the empty set and the original set itself, can be made from the set $\{6, 7, 8, 9, W, X, Y, Z\}$?

(2 marks)

b) Evaluate ${}^{50}P_3 \div {}^7P_2$.

(2 marks)

c) An excerpt of Pascal's triangle has been given, below right. Use this to evaluate $\binom{27}{3} - \binom{29}{27}$.

(2 marks)

	1	25	300
1	26	325	
1	27	351	2925
28	378	3276	
29	406	3654	23751
435	4060	27405	

Question 5 (4 marks)

For each of the following statements, state whether it is true or false, and give an example or counterexample to justify your conclusion. You do NOT need to complete any proof.

a) If an animal is a bird, then it can fly.

(2 marks)

b) All linear graphs have exactly two axes intercepts.

(2 marks)

Question 6 (5 marks)

Solve for n if $12 \binom{n}{2} = \binom{2n}{3}$. A trial and error method will not receive any marks.



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Test 1 – Proofs and Counting Techniques (10%)

2.3.1, 1.1.1, 1.1.3 – 1.1.5, 1.2.1 – 1.2.9

Time Allowed 20 minutes	First Name	Surname	Marks / 21 marks
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Circle your Teacher's Name:

Ms Chua

Mr Koulianos

Mr Liu

Ms Robinson

Assessment Conditions: (N.B. Sufficient working out must be shown to gain full marks)

- ❖ Calculators: Allowed
- ❖ Formula Sheet: Provided
- ❖ Notes: Not Allowed

PART B – CALCULATOR ASSUMED

Question 7 (4 marks)

a) What is the negation of the statement: $\forall x \in \mathbb{Z}, x^2 - 3x + 5$ is odd.

(2 marks)

b) Prove the following statement is true.

$$\exists a \in \mathbb{R} \text{ such that } \sqrt{a^3 - 4a + 1} \in \mathbb{Q}$$

(2 marks)

Question 8 (3 marks)

The local high schools want to form an interschool volleyball team of 6 by randomly selecting players from their top teams. The table below shows the number of players available in each school.

At least one student must be selected from each school, and there must be an equal number selected from Rossmoyne and Willetton. How many teams are possible?

School	Number of Players Available
Rossmoyne	12
Willetton	8
Applecross	5

Question 9 (4 marks)

Four of the letters from the word CATCHCRY are ordered. How many arrangements are possible?

Question 10 (4 marks)

Consider the set of integers between 1 and 500 inclusive.

How many of these integers are a multiple of 4, a multiple of 6 or a multiple of 7?

Question 11 (6 marks)

A group of trees are to be planted in a new local park: 3 eucalyptus, 2 golden wattle and 5 paperbark. Assume that all of the trees of the same type are indistinguishable from each other, e.g. all 3 of the eucalyptus trees can be considered the same.

- a) If the trees are to be planted in a single row, how many arrangements involve the golden wattle trees at the very ends of the row (i.e. one at each end)?

(1 mark)

- b) If the trees are to be planted in a single row, how many arrangements are there where the three eucalyptus trees are **not** all kept together?

(2 marks)

- c) If the trees are to be planted in two rows (where the back row has 6 trees, and the front row has 4 trees), how many arrangements can be made if the three eucalyptus trees must all be in the front row?

(3 marks)